9.1 Power Series

Infinite Series $\sum_{k=1}^{\infty} a_k$

Partial Sum $\sum_{k=1}^{n} a_k$

Does the series 3/10 + 3/100 + 3/1000 + ... converge or diverge?

(sum) $a = \frac{3}{10}$ $r = \frac{1}{10}$ $\frac{3/10}{1 - 1/10} = \frac{1}{3}$

sequence: $\frac{3}{10}, \frac{3}{100}, \frac{3}{1000}, \ldots$
(list)

$$\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \ldots = \frac{1}{3}$$

$$.3 + .03 + .003 + \ldots = .33\bar{3} = \frac{1}{3}$$

$$\sum_{n=2}^{\infty} 3\left(\frac{1}{10}\right)^{n-1} = \frac{1}{3}$$

Jan 25-11:33 AM

Infinite Geometric Series $\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \ldots = \boxed{\frac{a}{1-r}}$ $\boxed{\text{if } |r| < 1}$

Tell whether each series converges or diverges. If it converges, give its sum.

$$\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{n-1} = 3 + \frac{3}{2} + \frac{3}{4} + \ldots = \frac{3}{1 - \frac{1}{2}} = 6$$

$a = 3$ $r = \frac{1}{2}$

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots = \frac{2}{3}$$

$$\sum_{k=0}^{\infty} \left(\frac{3}{5}\right)^k = 1 + \frac{3}{5} + \frac{9}{25} + \ldots = \frac{1}{1 - \frac{3}{5}}$$

$a = 1$ $r = 3/5$

$$= \frac{1}{2/5} = \frac{5}{2}$$

Jan 25-11:38 AM

Find the function that is equal to the series. Support graphically.

$$1 + x + x^2 + x^3 + \ldots + x^n + \ldots = \frac{1}{1-x}$$

$a = 1$ $r = x$

if $|r| < 1$

$|x| < 1$

$-1 < x < 1$

Jan 25-11:45 AM

Power Series

centered at x=0: $\sum_{n=0}^{\infty} c_n x^n$

centered at x=a: $\sum_{n=0}^{\infty} c_n (x-a)^n$

Jan 25-11:48 AM

Find power series for the following functions

$$f(x) = \frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 \ldots$$

$a = 1,$ $r = -x$

$$f(x) = \frac{1}{x} = 1 - (x-1) + (x-1)^2 - (x-1)^3 \ldots$$

Hint: $\frac{1}{x} = \frac{1}{1 + (x-1)}$

$a = 1$ $r = -(x-1)$

Jan 25-11:55 AM

Differentiate the power series for $\frac{1}{1-x}$

What function equals this new power series?

$$\frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{d}{dx}\left[1 + x + x^2 + x^3 + \ldots\right]$$

$$\frac{1}{(1-x)^2} = 0 + 1 + 2x + 3x^2 + \ldots$$

Jan 25-11:58 AM

Use the power series for $\frac{1}{1+x}$ to find a power series for ln(1+x)

$$\int \frac{1}{1+x} = \int 1 - x + x^2 - x^3 + x^4 \dots$$

$a=1, r=-x$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \dots + C$$

Jan 25-12:00 PM

Find a power series for $\frac{1}{1+x^2}$

and use that series to find one for $\tan^{-1} x$

$$\int \frac{1}{1+x^2} = \int 1 - x^2 + x^4 - x^6 \dots$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} \dots + C$$

Jan 25-12:01 PM